2316 175

Class - BA/ BSc. I (Sem II)

Subject - Mathematics

Paper - (I) Calculus & Differential Equations

Time Allowed : 3 Hours Maximum Marks : 50

Note:- Attempt any five questions selecting at least two from each section.

1. (a) Find the interval in which the curve $y = e^x (\cos x + \sin x)$ is concave upawards or downwards in $(0, 2\pi)$. Find also the points of inflexion.

(b) Determine the position and nature of the double points on the curve

$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$ 5,5

2. (a) Find the equation of the cubic which has the same asymptotes as the curve $x^2y - xy^2 + xy + y^2 + x - y = 0$ and passes through the points (0,0), (1,0) and (0,1).

(b) Find the centre of curvature at any point (x,y) of the parabola $y^2 = 4ax$. Also find its evolution. 5,5

3. (a) Obtain a reduction formula for $\int x^m (\log x)^n dx$, $x > 0$; m, n natural numbers and hence evaluate

$$\int_0^1 x^4 (\log x)^3 dx.$$

(b) If $U_n \int_0^{\pi/2} x^n \sin x dx$ and $n > 1$ prove that

U_n (n – 1) $U_{n-2} = n \left(\frac{\pi}{2}\right)^{n-1}$ 5,5

4. (a) Determine by integration the limiting value of the sum of the following series when n is indefinitely great $\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n}$

(b) Find the length of the arc of the parabola $y^2 - 4y + 2x = 0$ which lies in the first quadrant. 5,5

Section-B

5. (a) Define exact differential equation and solve

$$xdx + ydy = \frac{a^2(xdy - ydx)}{x^2 + y^2}$$

(b) Solve $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)dy = 0$ 5,5

6. (a) Solve $(x^2 + y^2)(1 + p)^2 - 2(x + y)(1 + p)(x + yp) + (x + yp)^2 = 0$

(b) Find the complete solution and singular solution of $(a^2 - x^2)\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + b^2 - y^2 = 0$ 6,4

7. (a) Find the orthogonal trajectories of the curve

$$y = \frac{x^3 - a^3}{3x}$$, a being parameter.

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x \sin x \qquad 5,5$$

8. (a) Solve $(D^2 + 4)\, y = 4 \sec^2 2x$, by the method of variation of parameters.

(b) Solve the differential equation

$$x^3\frac{d^3y}{dx^3} + 6x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - 4y = (\log x)^2 \qquad 5,5$$

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